Non-Reducibility with Knowledge *wh*: 
Experimental Investigations

1 Knowing *wh* and Knowing *that*

- Obvious starting picture: (1) implies (2) (2) iff (3)
  
  1. John knows that he can buy an Italian newspaper at PaperWorld.
  2. John knows where he can buy an Italian newspaper.
  3. \( \exists x \text{ s.t. } J \text{ knows he can buy an Italian newspaper at } x. \)

- General case: (4) iff (5)
  
  4. J. knows [WH QUESTION]
  5. \( \exists p \text{ s.t.} \)
     
     \[ p \text{ is (a sufficient/a maximal/the best) answer to } [WH-Q] \]
     
     and J. knows that \( p \).

(RT) Reducibility Thesis:

Whether you know WH-Q depends *only* on which answers you know.

- RT is typical of theories of knowing *wh*.
- Why do we care about RT?
- RT plays well with important intuition:
  
  *know that* and *know wh* involve same know.
- RT account generalizes to other attitudes:
  
  agreeing, forgetting, certainty, ...
- If RT fails, we’ll want a new story for these connections.
2 False Beliefs & Non-Reducibility

- Claim: knowledge \( \textit{wh} \) doesn’t just depend on answers you know.
- It also depends on what false (potential) answers you believe.
- Claim contradicts RT.
- Instead, knowledge \( \textit{wh} \) reduces to knowledge \( \textit{that} + \text{false belief that} \).
- Non-reducible mention-some \( \text{George} \ (2011, 2013) \):
  + know one true item + don’t believe any false items.
- Non-reducible mention-all \( \text{Spector} \ (2005); \text{Cremers and Chemla} \ (2016) \):
  + know every (positive) true item + don’t believe any false items.
- Both just ask whether there are false beliefs; don’t look at how many.

3 George’s \(2011; 2013\) Judgment

- Reported judgment: \(6\) true in \text{AllTrue}, false in \text{Mixed}.

\begin{align*}
(6) & \quad \text{Sue knows where she can buy an Italian newspaper.} \\
\text{(AllTrue)} & \quad \text{Sue is standing on the street near a store called Newstopia. Sue’s friend, Bob, a native of the city who is normally very well-informed and trustworthy, told her that she can buy an Italian newspaper at Newstopia. Having no reason to doubt this, Sue took Bob at his word. Bob was correct about Newstopia, which does sell Italian newspapers.} \\
\text{(Mixed)} & \quad \text{Sue is standing on the street near two stores: one called PaperWorld, and another called Newstopia. Sue’s friend, Bob, a native of the city who is normally very well-informed and trustworthy, told her that she can buy an Italian newspaper at PaperWorld, and also at Newstopia. Having no reason to doubt this, Sue took Bob at his word. However, Bob completely misinformed Sue about PaperWorld, which does not sell newspapers, but actually just sells stationery and office supplies. Bob was correct about Newstopia.}
\end{align*}

\begin{tabular}{|c|c|c|}
\hline
 & \textbf{Newstopia sells Italian papers?} & \textbf{PaperWorld sells Italian Papers?} \\
\hline
Sue’s beliefs in \textit{AllTrue}: & YES & N/A \\
\hline
The facts in \textit{AllTrue}: & YES & N/A \\
\hline
The facts in \textit{Mixed}: & YES & NO \\
\hline
Sue’s beliefs in \textit{Mixed}: & YES & YES \\
\hline
\end{tabular}

This is a slight variation on George’s example, equivalent for purposes of the relevant theories.
• George claims [6] true in AllTrue, not in Mixed.
• Available RT accounts predict [6] has same truth value in AllTrue and Mixed.

- **Problem**: Judgment more fragile than optimal.
- **Problem**: Is apparently degraded truth a semantic/conventional matter, or truth-with-infelicity?
- **Problem**: Untrue how? Just false, or something more exotic?
- **Problem**: Proportions seem relevant.
  Available theories predict 1% false beliefs as bad as 99% false beliefs. Is this plausible?

4 An Initial Test

We began with a simple test of ordinary English speakers’ truth assessments of [6] in both AllTrue and Mixed.

4.1 Design

Participants read the vignettes AllTrue (in which Sue knows an answer and has only true beliefs) and Mixed (in which Sue knows an answer and has relevant false beliefs) in counterbalanced order. After reading each vignette, participants were asked to indicate their agreement with [6] on a scale from 1 (“Completely disagree”) to 7 (“Completely agree”).

[6] Sue knows where she can buy an Italian newspaper.

4.2 Results

- Participants tended to agree with [6] more strongly in AllTrue than in Mixed.

  \[ M = 6.23 \text{ vs. } M = 4.88, t(79.83) = 3.75, p < .001, d = .757 \]

- Additional analyses revealed that participants’ responses were not affected by the order in which they completed the study.

  No main effect of order, \( \chi^2(1) = 0.17, p = .682 \), and no interaction effect, \( \chi^2(1) = 2.51, p = .113 \).
4.3 The upshot

- Some initial evidence that ordinary truth value judgments distinguish AllTrue from Mixed.
- Also worth noting that participants’ level of agreement in the Mixed case also differs from what would be expected in a clearly false case.

5 A More Robust Test

We then replicated and expanded on this initial study in three ways:

- We included a comparison case in which the agent’s beliefs were all false.
- We included negated knowledge wh ascriptions.
- We looked at several related where questions, as in (7-a)–(7-c).

(7) a. Sue knows where she can buy an Italian newspaper.
b. Sue knows where to buy an Italian newspaper.
c. Sue knows where she should buy an Italian newspaper.
5.1 Design

Participants were randomly assigned to read AllTrue, Mixed, or AllFalse. (AllFalse) Sue is standing on the street near a store called Paperworld. Sue’s friend, Bob, a native of the city who is normally very well-informed and trustworthy, told her that she can buy an Italian newspaper at Paperworld. Having no reason to doubt this, Sue has always assumed that Bob was right. However, Bob completely misinformed Sue about Paperworld, which does not sell newspapers, but actually just sells stationery and office supplies.

After reading the vignette, participants were either asked to rate their agreement with (7-a)−(7-c) or were asked to rate their agreement with the negated form of one of these three knowledge wh ascriptions (8-a)−(8-c).

(8) a. Sue doesn’t know where she can buy an Italian newspaper.
b. Sue doesn’t know where to buy an Italian newspaper.
c. Sue doesn’t know where she should buy an Italian newspaper.

5.2 Results

Figure 2: Boxplots of participants’ agreement rating with the non-negated (left) and negated (right) knowledge ascriptions in the AllTrue, Mixed and AllFalse conditions for all three forms of knowledge wh statements.
First, consider participants’ agreement ratings with the the non-negated knowledge wh ascriptions:

- Participants agreed more in the AllTrue than in the Mixed conditions.
  
  $(M = 6.24)$ vs. $(M = 4.24)$, $t(213.87) = 10.00$, $p < .001$, $d = 1.285$.

- Participants agreed more in the Mixed than in the All false conditions.
  
  $(M = 4.24)$ vs. $(M = 1.97)$, $t(205.08) = 12.08$, $p < .001$, $d = 1.432$.

Next, consider participants’ agreement ratings with the the negated knowledge wh ascriptions:

- Participants agreed less in the AllTrue than in the Mixed conditions.
  
  $(M = 2.84)$ vs. $(M = 3.50)$, $t(219.52) = -2.41$, $p = .017$, $d = 0.322$.

- Participants agreed less in the Mixed than in the All false conditions.
  
  $(M = 3.50)$ vs. $(M = 5.66)$, $t(205.08) = 12.08$, $p < .001$, $d = 1.232$.

5.3 The upshot

- Robust evidence for an effect that supports non-reducibility.

- At the same time, Mixed clearly differs from All false, suggesting that we may want to try accounting for the effect without appealing to a semantic/conventionalized account.

6 The Naive Relevance-Implicature Approach

One natural worry to have is that we may have been unintentionally varying the relevance of the knowledge facts to the implied problem-at-hand (cf. Grice [1975]). We assessed this possibility by varying whether the person to whom knowledge is ascribed shares a language in common with the person looking for a newspaper.

If usefulness as a problem-solving resource is driving a relevance implicature effect responsible for the pattern of judgments, we should expect false beliefs and lack of a shared language to produce similar degraded truth judgments.

6.1 Design

We randomly assigned participants to read either (9), in which a woman named Sue and a man named Bob do not share a common language or (10), in which they do.

(9) **No Shared language**: Sue, who speaks only Italian and English, needs to buy an Italian newspaper. She is standing on the street near two stores: one called PaperWorld, and one called Cellulose City. Sue sees a man named Bob nearby. Bob is a native of the city who is normally very well-informed and trustworthy.
Bob speaks only Cantonese and Hungarian. He believes that Paper-World and Cellulose City sell Italian newspapers. He would be happy to tell this to Sue if she asked and he understood her. However, Bob would not be able to understand Sue’s question, since they can’t speak the same language.

(10) **Shared language:** Sue, who speaks only Italian and English, needs to buy an Italian newspaper. She is standing on the street near two stores: one called PaperWorld, and one called Cellulose City. Sue sees a man named Bob nearby. Bob is a native of the city who is normally very well-informed and trustworthy.

Bob speaks only English and Hungarian. He believes that PaperWorld and Cellulose City sell Italian newspapers. He would be happy to tell this to Sue if she asked and he understood her. Moreover, Bob should be able to understand Sue’s question, since they can speak the same language.

In each case, Bob’s beliefs about where to buy an Italian newspaper could have been **AllTrue** (11-a), **Mixed** (11-b), or **AllFalse** (11-c)

(11) a. Bob is right about both stores: PaperWorld and Cellulose City both sell Italian newspapers.

b. Bob is right about PaperWorld. However, Bob is mistaken about Cellulose City, which does not sell Italian newspapers. (It is actually a stationery shop.)

c. Bob is completely mistaken about PaperWorld and Cellulose City, neither of which sells Italian newspapers. (They are actually both stationery shops.)

Participants rated their agreement with (12) on a scale from 1 (‘Completely disagree’) to 7 (‘Completely agree’).

(12) Bob knows where Sue can buy an Italian newspaper.

Additionally, participants completed an item that tests whether we had been successful in manipulating the relevance-implicature (13)

(13) Because Sue needs an Italian newspaper, she calls her friend Mary, to ask her what she should do, and happens to mentions that she sees Bob nearby. Mary, who knows that Bob likes to keep up with the latest news from Italy, tells Sue

“Bob knows where you can buy an Italian newspaper.”

Given everything you know about Bob, how useful will Mary’s information be to Sue?

Participants answered on a scale from 1 (‘Not at all useful’) to 7 (‘Very useful’).
6.2 Results

First, consider participants’ answers to the usefulness question (13).

8 participants were excluded from the analyses because they indicated that English was not their native language. An additional 16 participants were excluded for failing to correctly answer the first comprehension question.

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Figure 3: Boxplots of participants’ ratings of the usefulness of the knowledge ascription.

- Participants found the knowledge ascription to be much less useful overall when Bob and Sue did not share a language than when they did.

  \[ F(1, 227) = 42.04, p < .001 \eta^2_p = .156 \]

- More importantly, when Bob and Sue did not share a language, participants’ usefulness judgments were not significantly affected by whether Bobs’ beliefs were AllTrue or Mixed.

  \[ (M = 3.93) \text{ vs. } (M = 3.46), t(73.34) = 0.991, p = .325, d = 0.222. \]

- This provides a test case for asking whether we still get a significant difference in the truth value judgments between these two cases.

We next considered participants’ truth value assessments of (12).

- Participants truth value judgments were not overall significantly affected by whether or not Sue and Bob shared a language.

  \[ F(1, 227) = 3.58, p = .060 \eta^2_p = .016 \]

- More importantly, when Bob and Sue did not share a language, participants’ usefulness judgments were significantly affected by whether Bobs’ beliefs were AllTrue or Mixed.

  \[ F(2, 227) = 256.17, p < .001 \eta^2_p = .693, \text{ and more importantly did not reveal a Language\times False Belief interaction effect, } F(2, 227) = 0.26, p = .773 \eta^2_p = .002. \]
Figure 4: Boxplots of participants’ agreement ratings with the knowledge \textit{wh} ascription.

\[(M = 6.63) \text{ vs. } (M = 5.21), t(49.34) = 4.89, p < .001, d = 1.111.\]

- This case provides a clear instance in which false beliefs are affecting participants’ truth value judgments in a way that cannot be explained by differences in the relevance of the knowledge \textit{wh} ascriptions.

6.3 The upshot

- These results count strongly against the relevance-implicature explanation of the non-reducibility effect.

- They do not, of course, rule out all possible pragmatic accounts, but we’ll set this issue to one side until some other specific proposal is offered.

7 Proportional Knowledge \textit{wh}

While the previous studies were informative in demonstrating \textit{that} false beliefs affect truth assessments of knowledge \textit{wh} ascriptions, they are not particularly informative as to \textit{how} false beliefs affect participants’ truth assessments.

We designed a final test that allowed us to come closer to parametrically varying the proportion of the agent’s beliefs that were false. This should allow us to begin measuring how participants’ truth assessments of knowledge \textit{wh} ascriptions vary as function of the increasing presence of false beliefs.

One possibility is that previous analyses were correct in providing a semantic/conventionalized account of the role for false beliefs in knowledge \textit{wh}, but wrong in the particulars of the role that they were given (see, George’s [2011, 2013]).
7.1 Design

All participants read a vignette in which a woman named Sue was told where to buy a newspaper by her friend Bob (14).

(14) Sue is standing on the street near three stores: one called PaperWorld, one called Cellulose City, and one called Newstopia. Sue’s friend, Bob, a native of the city who is normally very well-informed and trustworthy, told her that PaperWorld, Newstopia, and Cellulose City all sell Italian newspapers. Having no reason to doubt this, Sue has always assumed that Bob was right.

Participants were told that Bob was either correct about all three stores (15), correct about two of the three stores (16), correct about only one of the stores (17), or incorrect about all of the stores (18).

(15) Bob was correct about all three stores, which do in fact all sell Italian newspapers.

(16) However, Bob completely misinformed Sue about PaperWorld, which does not sell newspapers, but is actually a stationery shop. Bob was correct about Newstopia and Cellulose City.

(17) However, Bob completely misinformed Sue about PaperWorld and Cellulose City, which do not sell newspapers, but are actually both stationery shops. Bob was correct about Newstopia.

(18) However, Bob completely misinformed Sue about all three stores, none of which sell Italian newspapers. PaperWorld and Cellulose City, are actually both stationary shops, and Newstopia is an ironically misnamed shop that sells T-shirts with obnoxious political slogans.

After reading the vignette, participants rated their agreement with (6), as in previous studies.

(6) Sue knows where she can buy an Italian newspaper.

7.2 Results

• Participants’ agreement ratings were strongly affected by the proportion of the agent’s beliefs that were false.

\[ F(3, 164) = 52.74, \ p < .001 \ \eta^2 = .491 \]

• Participant’s judgments differentiated each of the four cases from each other.

\[ p’s < .05; \ d’s > .56 \]

• Additionally, in the 2 of 3 false beliefs condition, participants actually tended to disagree with the knowledge ascription.

\[ M(SD) = 3.77(2.01) \]
7.3 The upshot

- Truth assessments of knowledge \( wh \) ascriptions are sensitive not only to the presence of false beliefs but also to the proportion of the agent’s beliefs that are false.

- When the agent had proportionally more false than true beliefs, we observed overall disagreement with the knowledge \( wh \) ascription.

- These results motivate an account of knowledge \( wh \) with the resources to handle the sort of proportionality effect we observe here.

8 General Discussion

- What’s the lesson?
  It’s complicated.

- There is a false belief effect.

- This effect isn’t easily handled by obvious relevance implicature story.

- There are proportionality effects that nobody seems to predict.

Of course, this is hard to rule out entirely, and we’ve only provided a first step here.
9 One Candidate Account: Threshold/Standard-Sensitivity

- Old story: (19) true iff (20) is.

(19) John knows where he can buy an Italian newspaper.

(20) There is $x$ s.t. J. knows he can buy an Italian newspaper at $x$.

- George (2011, 2013) proposed: (19) true iff both (20) and (21) are.

(21) For every $x$ s.t. J. believes he can buy an Italian newspaper at $x$, he in fact can.

- New family of proposals: (19) true iff both (20) and \( (22) \) are.

(22) a. For many $x$ s.t. J. believes he can buy an Italian paper at $x$, he in fact can.
   (Where the amount that constitutes many is in part determined by conversational context.)

b. For enough $x$ s.t. J. believes he can buy an Italian paper at $x$, he in fact can.
   (Where the amount that constitutes enough is in part determined by conversational context.)

c. For $\geq n\%$ of the $x$ s.t. J. believes he can buy an Italian paper at $x$, he in fact can.
   (Where $n$ is a contextually supplied threshold.)

- Coupled with a theory where different speakers can have different ideas about threshold, and where threshold-vagueness-induced truth value uncertainty results in intermediate truth judgment reports, this plausibly derives gradability effect.

- More exploration required.

- Above approach assumes more-or-less bivalent semantics - gradable truth would provide another set of tools.
A A Digression on Knowledge *that*

It seemed to us intuitively clear that the key knowledge *that* claims are simply true in both the **AllTrue** and **Mixed** scenarios. However there is, in general, reason to think that false beliefs about related propositions can sometimes render otherwise good knowledge *that* ascriptions untrue (cf. [Goldman (1976)](#)). To check our intuition, we conducted an experiment to assess the truth of knowledge *that* claims in the situations described by vignettes **AllTrue**, **Mixed**, and **AllFalse**.

A.1 Design

The methods for this study are similar to those in Section 5 except that participants were instead asked to rate their agreement with knowledge *that* ascriptions. In the cases where all of Sue’s beliefs were true and where Sue’s beliefs were both true and false, participants rated their agreement with (23-a). Participants who read the case in which Sue’s belief was simply false instead rated their agreement with (23-b) since the this vignette did not mention Newstopia.

(23)  

a. Sues knows that she can buy an Italian newspaper at Newstopia.  

b. Sue knows that she can buy an Italian newspaper at Paperworld.

After rating their agreement, participants were asked to provide a summary of what they read and then completed series of demographic items including a question asking whether English was their native language.

A.2 Results

An analysis of the remaining participants’ agreement ratings with (23-a) revealed that there was no significant difference in their agreement ratings when Sue had only true beliefs ($M(SD) = 5.94(1.22)$) and when Sue had both true and false beliefs ($M(SD) = 5.64(1.86)$), $t(71.01) = 0.830$, $p = .409$, $d = 0.184$. For comparison, participants did agree significantly disagree with (23-b) (i.e., with a mean agreement rating below 4) when Sue had only false beliefs ($M(SD) = 2.86(1.99)$), $t(82) = −5.244$, $p < .001$ (Fig. 6).

A.3 The upshot

While we saw above in Sections 4 and 5 that false beliefs reduce truth judgments for knowledge *wh* ascriptions, we find no analogous effect for knowledge *that* ascriptions. This suggests that we have a genuine challenge to reductive accounts: the different scenarios support the same facts regarding knowledge *that*, suggesting that knowledge *that* does not suffice to determine knowledge *wh*. 
Figure 6: Boxplots of participants' agreement rating with the knowledge that ascription in the AllTrue, Mixed and AllFalse conditions.

References


