Rectification and precession signals in the climate system

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[1] Precession of the equinoxes has no effect on the mean annual insolation, but does modulate the amplitude of the seasonal cycle. In a linear climate system, there would be no energy near the 21,000 year precession period. It is only when a non-linear mechanism rectifies the seasonal modulation that precession-period variability appears. Such rectification can arise from physical processes within the climate system, for example a dependence of ice cover only on summer maximum insolation. The possibility exists, however, that the seasonality inherent in many climate proxies will produce precession-period variability in the records independent of any precession-period variability in the climate. One must distinguish this instrumental effect from true climate responses. Careful examination of regions without seasonal cycles, for example the abyssal ocean, and the use of proxies with different seasonal responses, might permit separation of physical from instrumental effects. *INDEX TERMS:* 4267 Oceanography: General: Paleoceanography; 3344 Meteorology and Atmospheric Dynamics: Paleoclimatology; 4215 Oceanography: General: Climate and interannual variability (3309). **Citation:** Huybers, P., and C. Wunsch, Rectification and precession signals in the climate system, *Geophys. Res. Lett.*, *30*(19), 2011, doi:10.1029/2003GL017875, 2003.

1. Introduction

[2] One of the most important elements in the discussion of climate change concerns the appearance in, and possible dominance by, Milankovitch cycles in paleoclimate records. Setting aside the 100 kyr band, whose relationship to Milankovitch forcing remains problematic [e.g, *Roe and Allen*, 1999], the Milankovitch-forced energy is largely, but not wholly, contained within two bands around 41 kyr and 21 kyr—the obliquity and precessional bands respectively [*Bradley*, 1999; *Cronin*, 1999].

[3] In particular, reports of strong precessional signals in various records are widespread; among the most recent reports are *Lamy et al.* [1998] for deep-sea sediments, *Thamban et al.* [2002] for monsoon strength, and *Bozzano et al.* [2002] for atmospheric dust. Such signals are usually interpreted as demonstrating orbital-period climate variability [e.g., *Ruddiman and McIntyre*, 1981; *Imbrie et al.*, 1992]. Here we raise the question of whether these signals are due to subannual climate variability or, at least in part, are an artifact of the way in which climate signals are recorded.

2. Obtaining Precessional Rectification

[4] Changes in Earth's obliquity alter the amplitude of the seasonal cycle and generate low-frequency shifts in the latitudinal distribution of insolation. Precessional changes also alter the seasonal cycle, but in contrast to obliquity, cause no change in annual average insolation at any latitude [*Rubincam*, 1994]. A general expression for insolation contains terms related to seasonal variability of the form,

$$\begin{aligned} \mathcal{F} &= a \sin\epsilon \sin M + b \sin(M - \varpi) + \ldots \\ &\equiv \mathcal{F}_1 + \mathcal{F}_2 + \ldots . \end{aligned} \tag{1}$$

Here, M is the true anomaly, an angle increasing by 360° per year, ϵ is the obliquity, varying between 22° and 25° with a time scale of about 41 kyr; and ϖ is the angle between perihelion and the vernal equinox and varies with periods dominantly between 19 and 23 kyr. a, b are coefficients that are either constant or have even lower frequency dependencies.

[5] Both terms $\mathcal{F}_{1,2}$ vary at periods of close to one year. $\mathcal{F}_1$ has an annual carrier frequency, $s_a = \dot{M}/2\pi$, the dot denoting the time derivative, and is amplitude modulated by obliquity at a frequency $s_\epsilon = \dot{\epsilon}/2\pi$. The amplitude modulation involves two combination frequencies $s_a \pm s_\epsilon \approx s_a$, which vanish when averaged over a tropical year. In $\mathcal{F}_2$, the frequency is $s_a - s_\varpi \approx s_a$; because $s_\varpi \ll s_a$, the forcing averages to zero over any integral multiple of durations $2\pi/(s_a - s_\varpi)$, that is over one anomalistic year. In the full insolation forcing ϵ also occurs independent of M, thus varying at low-frequencies, while all instances of ϖ appear in combination with M, thus varying at periods near one year.

[6] How does one obtain a low frequency response to high frequency insolation variations? There are several possibilities. Suppose, following the very large literature on Milankovitch forcing, that the climate system responds primarily to summer insolation. That is, simplifying slightly, let the climate system respond only when $\mathcal{F}$ is above some threshold, τ,

$$\begin{aligned} \mathcal{F}_r &= |\mathcal{F}|^{\nu}, \quad \tau \le \mathcal{F} \\ &= 0, \quad \text{otherwise} \end{aligned} \tag{2}$$

The effect of Equation (2) on $\mathcal{F}$ is an example of what is called a νth-power-law device [*Davenport and Root*, 1958;

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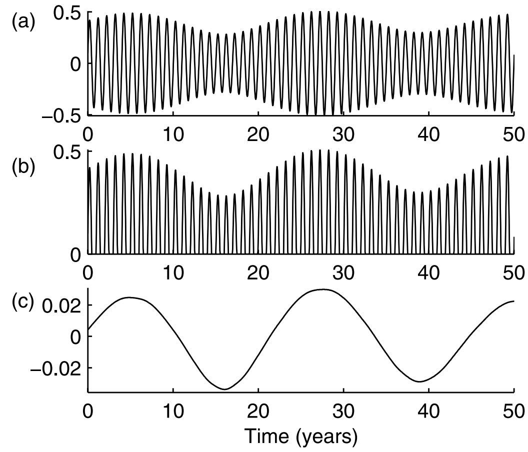

Figure 1. Production of low-frequency variability. (a), Simple amplitude-modulated signal of form (1) having no low frequency content. (b), Rectified signal according to (2) and then, (c), low pass filtered to leave only the envelope function. For visual clarity, the periods of the secular orbital terms are decreased by a factor of 1000 giving roughly 1/23 precession and 1/41 obliquity cycles per annual cycle.

Middleton, 1960]. General nonlinearities can be represented by superposition of devices with differing values of ν.

[7] A simple example is given by taking $\tau = 0$, $\nu = 1$, which is a "half-wave rectifier" or "detector" [e.g., *Zimmerman and Mason*, 1959]; an example of its effects can be seen in Figure 1. The simple supposition that only positive values are important immediately, and drastically, changes the frequency content of the forcing. Figure 2 displays the periodogram of forcings (1) and (2). $\mathcal{F}_o$ has no energy below the annual cycle, while the rectified signal $\mathcal{F}_r$ does. We will call this "climate-system rectification" and there are many physical processes which can act this way [e.g., *Kim et al.*, 1998; *Clement et al.*, 2000].

[8] So far there is nothing new here. But consider that exactly the same low frequency effect can be produced by the recording devices. These recorders can represent anything that has a seasonality, including foraminifera that grow only during one season or month, or just grow more in summer than in winter, or a tracer laid down by a windfield direction confined primarily to one month or season. (Rectification of the annual cycle is not the same as its aliasing [*Wunsch*, 2000], which is a result of discrete sampling. Purely analogue devices, such as ordinary radio receivers, employ rectifiers.) That is to say, the most obvious representation of a seasonal growth, wind, or precipitation dependence in tracers or organisms will be the same form as Equation (2).

[9] At least some of the inferred precessional signals are thus likely an artifact of seasonal biases in growth, wind, or temperature patterns, among other possibilities. Any recording medium, be it biological or physical, subject to an annual cycle, has to be examined for such rectification effects, and which could actually dominate the observed signals.

3. A More Complete Discussion

[10] General analytical expressions, involving hypergeometric functions, are available for the response of rectifiers to a variety of inputs [*Davenport and Root*, 1958; *Middleton*, 1960]. Because there are many terms in $\mathcal{F}$, however, a discussion of its rectification is more complicated than can be obtained by examining only one or two carrier frequency contributions, and it is simpler to compute the results numerically. We therefore use estimates of the secular variability in Earth's orbital parameters [*Berger and Loutre*, 1992] along with a numerical code to estimate mean diurnal insolation (*J. Levine*, personal communication, 2003) at 65°N over the last 800 kyr. This representation is incomplete at the highest frequencies—not including diurnal variations nor other very high-frequency perturbations. It is adequate nonetheless, to illustrate the influence of rectification on the annual cycle.

[11] Owing to the vastly different periods between the annual variability and the secular modulating terms, it is impractical to plot the full time series of insolation over timescales of interest. Instead, Figure 3 shows insolation at 65°N plotted at the equinoxes and solstices. The date of the solstices and autumnal equinox, assuming the vernal equinox is fixed at March 20th, can vary substantially [*Vernekar*, 1972]. Over the last 1000 kyr, for example, the autumnal equinox occurred between September 5th and October 1st, depending on Earth's mean radial velocity, or equivalently, the eccentricity and phase of precession. The magnitude of equinoctial insolation depends only on eccentricity and precession, whereas solstice insolation at high-latitudes is also influenced by obliquity. The variability in the date and magnitude of these snapshots of mean diurnal insolation are indicative of the phase and amplitude modulation of the full annual cycle.

[12] Application of the rectification device (2) to the insolation signals dramatically alters the low-frequency

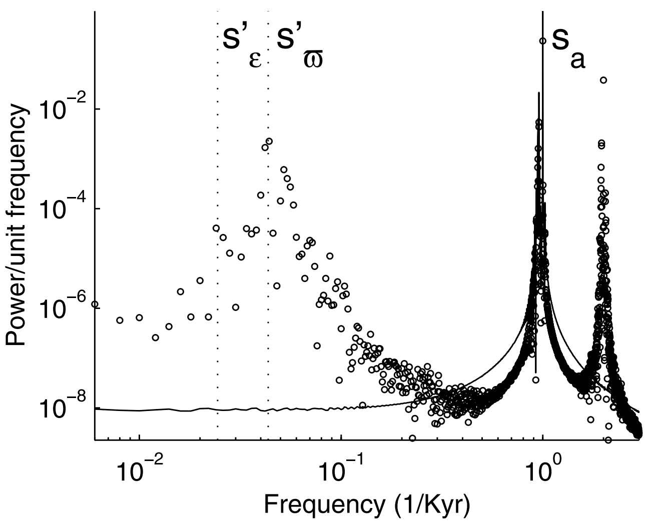

Figure 2. Periodograms of the original and rectified forcings. Solid line is from the original forcing (1) plus a small amount of white noise. The energy near the annual cycle, s_a, is split owing to modulation by the precession and obliquity terms, but there is no excess energy at the lower frequencies. Circles are the result after applying a half-wave rectifier to the signal. Now excess energy appears at the higher harmonics of s_a as well as the frequencies s'_ϖ and s'_ϵ where the primes indicate that the orbital terms have a 1000 fold decrease in period.

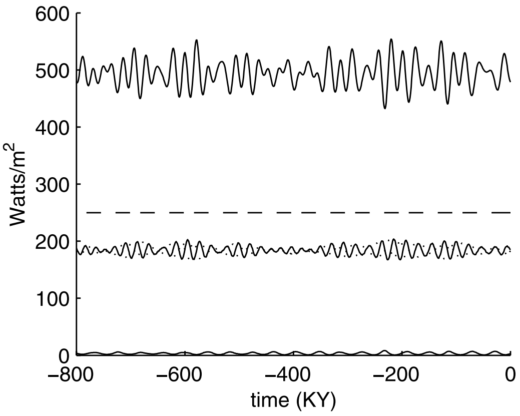

Figure 3. Mean diurnal insolation at 65°N. The full timeseries, sampled at 30 day intervals, oscillates too rapidly to be usefully plotted; instead snapshots of the insolation at the solstices and equinoxes are shown. Uppermost solid line is for the summer solstice, middle solid line is for the autumnal equinox, and near-zero solid line is at the winter solstice. The dotted line indicates the vernal equinox insolation. A similar plot appears in *Imbrie et al.* [1993], but there the vernal equinox and solstices are incorrectly assigned fixed dates. Horizontal dashed line indicates the lower level at which rectification is applied, denoted τ in Equation (2).

content of the insolation record. (Figure 4) shows results using τ = 250 Watts/m^2 and ν = 1, where the parameters are largely arbitrary. Other choices of τ and ν would change the distribution of energy in the rectified signal, but the basic effect—transferring energy from the high to low frequencies—is robust. Apart from the concentration of energy in the obliquity and precession bands, the rectified insolation also has enhanced energy in a broad-band ranging from millennial to 100 kyr periods. One source of this energy appears to be interactions between the modulation terms; another is the presence of low-frequency obliquity energy which, after rectification, is transferred into higher harmonics. The second harmonic of obliquity, 2/41 kyr, lies within the precession-band (Huybers and Wunsch, A depth-derived Pleistocene age-model: Uncertainty estimates, sedimentation variability, and nonlinear climate change, submitted for publication, 2003) thus providing another potential source for precession-band energy.

4. Further Considerations

[13] Another small rectification effect exists for insolation. In Figure 3 it is evident that winter solstice insolation variations are attenuated as compared with those of the summer solstice. Above the Arctic or Antarctic circles, attenuation becomes ‘‘clipping’’ as insolation goes to zero during polar night. This polar clipping is a form of rectification and is solely due to geometry. The effects account for the higher harmonics in the insolation cycle shown in (Figure 4), and the very slight excess in energy in the precession band. At higher latitudes, the geometric rectification is more pronounced, and (Figure 4) shows a periodogram of the low-frequencies in insolation at 85°N calculated over the last 800 kyr. Concentrations of energy are apparent in both the obliquity and precession bands. Geometrical rectification is also expected for the diurnal cycle, but we do not consider this higher frequency variability here.

[14] Suppose a component of the apparent signal arises from the recorder rectification with amplitude a and in-phase with the precession angle, written as $x_1(t) = a\cos(\varpi)$; suppose too, that the climate system itself produces a rectified signal with phase, η, which is faithfully reproduced

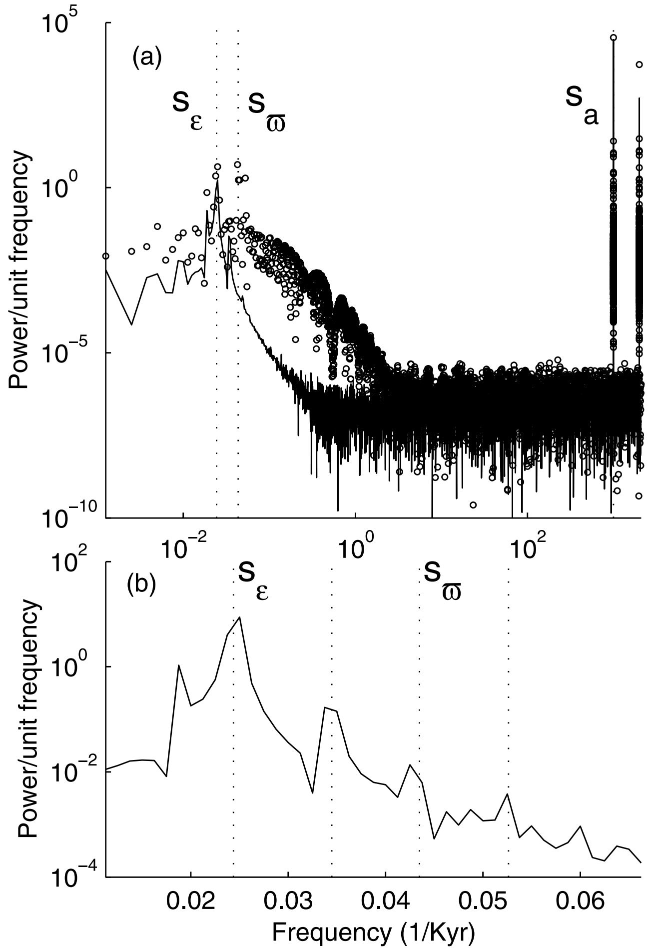

Figure 4. Periodograms of mean diurnal insolation plus a small amount of white noise. (a) Solid line is from insolation at 65°N, while circles are from insolation passed through a νth-law device with τ = 250 Watts/m^2 and ν = 1. After rectification, low frequency energy at the obliquity band (s_ϵ) is enhanced, and energy at the precession band (s_ϖ) now appears. The ordinate and abscissa are logarithmic. For plotting purposes, an exponentially diminishing number of periodogram estimates are shown for frequencies above 1/10 kyr except near the annual cycle and its first harmonic where full resolution is used—no significant structural changes result. (b) Periodogram of insolation at 85°N. Vertical lines from left to right are centered on the obliquity bands at 1/41 and a minor side-band at 1/29 kyr [*Melice et al.*, 2001] and precession at 1/23 and 1/19 kyr. The abscissa is linear, and for visual clarity, only the low-frequencies are shown. The seasonal cycle, s_a, is so much more powerful than any other insolation frequency (other than the diurnal) that its rectification is of greatest concern, but all frequencies are susceptible to such effects.

in a core record as $x_2(t) = b\cos(\varpi - \eta)$. Then omitting any stochastic component, the apparent signal at the precession frequency is,

$$x(t) = a\cos(\varpi) + b\cos(\varpi - \eta)$$
$$= \left(a^2 + b^2 + 2ab\cos\eta\right)^{1/2}$$
$$\times \cos\left(\varpi - \tan^{-1}\{b\sin\eta/(a + b\cos\eta)\}\right), \quad (3)$$

and one faces the problem of separating the recorder-rectified signal from that of the climate system. If another source is present due e.g., to geometrical rectification or higher harmonics of the obliquity energy, one has to separate a three-component vector sum.

[15] There is one medium, the deep ocean (below about 300m, with the major exception of the equator) that typically displays no sign of seasonal signals in velocity, temperature, or salinity. Measured variables reflecting only these physical processes, nonetheless having significant precessional-band signals, have a straightforward interpretation as showing rectification of the climate system, rather than that of the recording devices.

[16] The possibility of instrumental rectification renders the discussion of the relationship of proxies to climate variables a somewhat intricate one. In particular, one must carefully define ''climate'' change. Consider for example an earth in which hotter summers gave rise to a corresponding increase in precipitation, *P*. Suppose further that the increased *P* was exactly compensated by increased evaporation, *E*, during the colder winters. Then the anomaly of $P - E$ vanishes in the annual average, and there is no net climate change at low frequencies. Now suppose that increased precipitation and temperatures also lead to an increase in leaf mass of deciduous trees during the growing season and that all such leaves were shed during the autumn. Then a proxy based upon the annual mass of leaf generation would be rectified by the autumn shedding, and there would be a signal in the precession band that would be an incorrect measure of the annual average $P - E$. To the contrary however, if *P*, or *E*, by themselves are of interest, then the rectified leaf signal directly measures their low frequency content. Furthermore, leaf mass, with its influence on albedo and evapotranspiration, is itself a climate variable, and the rectified leaf-mass signal could itself be regarded as real climate change. Evidently, one must specify in detail the particular physical variable that the proxy is intended to represent before it can be interpreted.

5. Conclusion

[17] Our central point is that any precession-band energy appearing in climate time series requires the existence of a seasonal-cycle rectifier, and such rectifiers appear not only in the climate system itself, but also in the recording devices, both biological and physical. A similar phenomenon exists for the obliquity band, but analyzing this effect is more complex because obliquity band energy is also present in the forcing itself. To understand the origins of Milankovitch band energy in the climate record, one must apparently model the seasonal cycle in the recording instruments and correct for it in the climate variables.

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References

Berger, A., and M. F. Loutre, Astronomical solutions for paleoclimate studies over the last 3 million years, *Earth Planet. Sci. Lett.*, *111*, 369–382, 1992.

Bozzano, G., H. Kuhlmann, and B. Alonso, Storminess control over African dust input to the Moroccan Atlantic Margin (NW Africa) at the time of maximal boreal summer insolation: A record of the last 220kyr, *Palaeogeog. Palaeoclim. Palaeoecol.*, *183*, 155–168, 2002.

Bradley, R. S., *Paleoclimatology*, Academic, San Diego, 1999.

Clement, B., R. Seager, and M. Cane, Suppression of El Nino during the mid-Holocene by changes in the Earth's orbit, *Paleoceanogr.*, *15*, 731–737, 2000.

Cronin, T. M., *Principles of Paleoclimatology*, Columbia Univ. Press, New York, 1999.

Davenport, W. B., and, W. L. Root, *An Introduction to the Theory of Random Signals and Noise*, McGraw-Hill, New York, 1958.

Imbrie, J., A. Berger, and N. J. Shackleton, Role of orbital forcing: A two million year perspective, in *Global Changes in the Perspective of the Past: Dahlem Workshop Reports*, edited by J. A. Eddy and H. Oeschger, vol. 12, pp. 263–277, John Wiley and Sons Ltd, New York, 1993.

Imbrie, J., et al., On the structure and origin of major glaciation cycles. 1. Linear responses to Milankovitch forcing, *Paleoceanogr.*, *6*, 205–226, 1992.

Kim, S., T. Crowley, and A. Stossel, Local orbital forcing of Antarctic climate change during the last interglacial, *Science*, *280*, 728–730, 1998.

Lamy, F., D. Hebbeln, and G. Wefer, Late Quaternary precessional cycles of terrigenous sediment input off the Norte Chico (27.5°S) and palaeoclimate implications, *Paleogeog. Palaeoclim., Palaeoecol.*, *141*, 233–251, 1998.

Melice, J., A. Coron, and A. Berger, Amplitude and frequency modulation of the Earth's obliquity for the last million years, *J. Climate*, *14*, 1043–1054, 2001.

Middleton, D., *An Introduction to Statistical Communication Theorym*, McGraw-Hill, New York, 1960.

Roe, G., and M. Allen, A comparison of competing explanations for the 100,000-yr ice age cycle, *Geophys. Res. Lett.*, *26*, 2259–2262, 1999.

Rubincam, D., Insolation in terms of Earth's orbital parameters, *Theor. Appl. Climatol.*, *48*, 195–202, 1994.

Ruddiman, W. F., and A. McIntyre, Oceanic mechanisms for amplification of the 23,000-year ice-volume cycle, *Science*, *212*, 617–727, 1981.

Thamban, M., V. P. Rao, and R. R. Schneider, Reconstruction of late Quaternary monsoon oscillations based on clay mineral proxies using sediment cores from the Western Margin of India, *Mar. Geol.*, *15*, 417–424, 2002.

Vernekar, A. D., *Long-period variations of incoming solar radiation*, Am. Meteorol. Soc., Boston, Mass., 1972.

Wunsch, C., On sharp spectral lines in the climate record and the millennial peak, *Paleoceanog.*, *15*, 417–424, 2000.

Zimmerman, H., and S. J. Mason, *Electronic Circuit Theory*, Wiley, New York, 1959.

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